1 Introduction

Several features of supermassive black holes (or galactic black hole candidates) have been interpreted as evidence for black hole spin:


2. The ratio $R$ of observed quasar radiative energy per unit comoving volume to the current mass density of black holes is directly related to the mean radiative efficiency $\epsilon_M$ of accreting onto black holes (Soltan 1982): $\epsilon_M \geq R$. Recent measurements suggest $0.1 \lesssim R \lesssim 0.2$.

3. QPOs (quasi-periodic)

A physical or phenomenological model for the QPO can provide more stringent constraints but requires additional assumptions.

GRO J1655-40, 95% confidence limits on the mass:

$$v \sin i = \frac{K_2(1 + q)0.49q^{2/3}}{0.6q^{2/3} + \ln (1 + q^{1/3})}$$

(Horne, Wade, & Szkody 1986) and using the following relation

$$\frac{PK^3}{2\pi G} = \frac{M_1 \sin^3 i}{(1 + q)^3}$$

Then $M_1$ and $M_2$ to lie in the range 5.5-7.9$M_\odot$ and 1.7-3.3$M_\odot$ (Shahbaz et al. 1999).

Furthermore this result requires $a_\ast \gtrsim 0.15$ (Strohmayer 2001)

4. The shape of the X-ray continuum from an accreting black hole may depend on the spin.

Nonprimodial black holes form from gravitational collapse and in general are born with nonzero spin.

Heger et al. 2002: If the initial collapse occurs from a massive star, the spin depends on the angular momentum profile of the progenitor star and the (magneto)hydrodynamics of core collapse in evolved, spinning stars. Detailed Newtonian simulations of the collapse of spinning stars with $M \lesssim 300M_\odot$ have been performed and suggest how spinning black holes may arise during core collapse and the fate of the collapse depends drastically on the mass, spin, metallicity, and magnetic field of the progenitor, as well as details of the EOS and neutrino transport.

Assume that the orientation of the spin vector is fixed.

$$\dot{J} \equiv \left[ \int_{\text{on horizon}} d\theta d\phi \sqrt{-g} T^{r\phi} \right]$$

$$\dot{E} \equiv \left[ \int_{\text{on horizon}} d\theta d\phi \sqrt{-g} T^{r_t} = \dot{M} \right]$$

$$\dot{M}_0 \equiv \left[ \int_{\text{on horizon}} d\theta d\phi \sqrt{-g} \rho_0 u^r \right]$$

Then the dimensionless spin-up parameter $s$ can be defined

$$s \equiv \frac{d\alpha}{dt} \frac{M}{M_0}$$

$K_2 = 215.5 \pm 2.4 \text{km s}^{-1}$ and $v \sin i = 82.9 - 94.9 \text{km s}^{-1}$
Here let $\epsilon_M$ be the efficiency of conversion of rest-mass energy to luminous energy by accretion onto a black hole of mass $M$, and $\epsilon_L$ be the efficiency of accretion luminosity

$$\epsilon_M = \frac{L}{M_0 c^2}$$  \hspace{1cm} (1.3)

$$\epsilon_L = \frac{L}{L_E}$$  \hspace{1cm} (1.4)

where $L$ is the luminosity and $L_E$ is the Eddington luminosity, given by

$$L_E = \frac{4\pi M \mu_e m_p c}{\sigma_T} \approx 1.3 \times 10^{44} \mu_e M_8 \text{ erg sec}^{-1}$$  \hspace{1cm} (1.5)

Any contribution of collisionless or self-interacting dark matter may be negligible, and the accretion is assumed to be dominated by normal, baryonic matter, in particular, consisting of fully ionized atoms and the principal opacity source is to be Thomson scattering. The black hole growth rate must account for the loss of accretion mass energy in the form of outgoing radiation according to

$$\frac{dM}{dt} = (1 - \epsilon_M) \dot{M}_0$$  \hspace{1cm} (1.6)

The characteristic accretion time scale is defined as

$$\tau_{\text{acc}} = \frac{M c^2}{L_E} \approx 0.45 \mu_e \text{ Gyr}$$  \hspace{1cm} (1.7)

and is independent of $M$. Then the evolution equations of the black hole’s mass energy and spin are

$$\frac{dM}{dt} = \epsilon_L (1 - \epsilon_M) \frac{M}{\tau_{\text{acc}}}$$  \hspace{1cm} (1.8)

$$\frac{da_*}{dt} = \epsilon_L \frac{s}{\epsilon_M \tau_{\text{acc}}}$$  \hspace{1cm} (1.9)

disk model Determining $\epsilon_M(a_*)$ and $s(a_*)$ requires a gas dynamics model for black hole accretion.

(1) a standard, relativistic, Keplerian “thin disk” with “no-torque boundary condition” at the innermost stable circular orbit (ISCO).

(2) a relativistic, MHD accretion disk that accounts for the presence of a frozen-in magnetic field in a perfectly conducting plasma.

The torus in threaded with a poloidal magnetic field initially and evolves with an adiabatic EOS with $\Gamma = 4/3$ (to model a radiation-dominated, inner-motion EOS). The MRI instability - viscosity

The numerical simulations: in steady state the radiation efficiency parameter $\epsilon_M(a_*)$ is remarkably close to the function characterizing the standard thin disk, even though there is no sharp transition in the surface density at or near the ISCO. The spin evolution parameter $s(a_*)$ is different$^{3}$.

$$\epsilon_M = 1 - \tilde{E}_{\text{ISCO}}$$  \hspace{1cm} (1.10)

$$s = \tilde{L}_{\text{ISCO}} - 2a_* \tilde{E}_{\text{ISCO}}$$ \hspace{1cm} (standard thin disk)  \hspace{1cm} (1.11)

$$s = 3.14 - 3.30a_*$$ \hspace{1cm} (MHD disk)  \hspace{1cm} (1.12)

where $\tilde{E}_{\text{ISCO}}, \tilde{L}_{\text{ISCO}}$ are the energy and angular momentum of a unit mass at the ISCO

$$\tilde{E}_{\text{ISCO}} = \frac{r_{\text{ns}}^2 - 2Mr_{\text{ns}} + a\sqrt{Mr_{\text{ns}}} + a^2}{r_{\text{ns}}^2 - 3Mr_{\text{ns}} + 2a\sqrt{Mr_{\text{ns}}}}$$  \hspace{1cm} (1.13a)

$$\tilde{L}_{\text{ISCO}} = \frac{\sqrt{Mr_{\text{ns}}} (r_{\text{ns}}^2 - 2a\sqrt{Mr_{\text{ns}}})}{r_{\text{ns}}^2 - 3Mr_{\text{ns}} + 2a\sqrt{Mr_{\text{ns}}}}$$  \hspace{1cm} (1.13b)

and the coefficient of (1.12), $s/a_*$ is determined numerically (McKinney & Gammie 2004).

Assuming that both the mass and luminosity efficiencies remain constant with time, from (1.8) yielding

$$\frac{M(t)}{M(t_0)} = \exp \left[ \frac{\epsilon_L (1 - \epsilon_M) t - t_0}{\epsilon_M \tau_{\text{acc}}} \right]$$  \hspace{1cm} (1.14)

Plausible processes

1. Major mergers: a nearly equal mass BH-BH mergers Pfeiffer, Teukolsky, & Cook 2000

$$\frac{m_f^2}{M_{\odot} f^2} \leq 1 + \frac{(J/\mu m)^2}{4[1 + \sqrt{1 - (S/M_f)^2}]}$$  \hspace{1cm} (1.15)

For the $-0.50$ sequence $a_* \leq 0.92$, for the $+0.17$ sequence $a_* \leq 0.97$.

2. Minor mergers

Accretion of small companions with isotropically distributed orbital angular momenta results in spin-down, with $a_* \sim M^{-7/3}$ (Hughes & Blandford 2003)

$^3$De Villiers et al. 2004 used a different numerical method and took $\Gamma = 5/3$, resulting in the same relation.
3. Accretion: Fully relativistic MHD accretion simulations: the black holes that have grown primarily through accretion are not maximally rotating initial conditions:
Fishborne & Moncrief torus with inner radius at \( r = 6M \) and pressure maximum at \( r = 12M \)
The initial magnetic field is purely poloidal

While the specific energy accreted material is accurately predicted by the thin-disk model, the specific angular momentum is substantially lower: This is a result of ordered magnetic fields in the plunging region, which transport angular momentum outward into the bulk of the disk.

This sequence of accretion models reaches equilibrium for \( a_* \simeq 0.93 \) (Not suggesting that spin equilibrium is always reached at \( a_* \simeq 0.93 \)). Thinner disks imply lower accretion rates, so thin-disk accretion will have lower weight in determining the black hole spin than thick-disk accretion over a comparable timescale.

**SDSS 1148+5251** The redshift of a quasar discovered to date is \( z_{\text{QSO}} = 6.43 \) or \( t = 0.87 \text{Gyr} \) (Fan et al. 2003). — the most conservative hypothesis is that the seed black holes that later grow to become SMBHs originate from the collapse of Population III stars (Madau & Rees 2001).

Newtonian simulations suggest that the Pop. III stars with masses in the range \( M \sim 60 - 140M_\odot \) and \( M \gtrsim 260M_\odot \) collapse directly to black holes

\[ \text{‡} \]

The upper limit \( M \lesssim 600M_\odot \).

Assume that the duration of a merger, as well as the time required for accretion to drive the merged remnant to spin equilibrium, are both much shorter than the time interval between mergers, and that the hole continues to accrete steadily throughout this interval.

Black hole mergers can completely eject black holes from halo centers owing to gravitational wave recoil and thereby turn off accretion altogether

\[ \text{‡} \]

The stars with \( M \sim 140 - 260M_\odot \) undergo explosive annihilation via pair-creation processes
Then total mass amplification from amplification factor due to mergers, the hole, eventually forcing their mutual alignment but on a timescale that is still uncertain.

A major merger between two black holes of comparable mass may change both the magnitude and direction of the spin of the resulting black hole remnant. Following such a merger, the orientation of the black hole spin may not be aligned with the orientation of the asymptotic gaseous disk at radii \( r \gg 100M \) outside the hole. Moreover, the orientation of the asymptotic disk will likely fluctuate in time because of the redistribution of gas following mergers of dark halo cores, galaxy mergers, and the tidal disruptions of passing stars by the central hole.

Near the black hole, at radii from \( r \sim M \) to \( 20M \), where the bulk of the disk’s gravitational energy is released and the hole–disk interactions are strong, the hole’s gravitomagnetic field will exert a force on the disk that, when combined with viscous forces and magnetic fields, will drive the disk down into the hole’s equatorial plane: “Bardeen & Petterson effect”

The increase in mass by accretion at the Eddington limit is considerably smaller than \( t_{\text{accrete}} \).

Hence, \( t_{\text{QSO}} = 0.87 \)Gyr is the upper limit to the time available for accretion to occur onto the initial seed black hole that powers this quasar.

### 2 Black hole growth and spin-up

The increase in mass by accretion at the Eddington limit \( \epsilon_L = 1 \)

Evaluating \( t(z) \) with the WMAP values: \( \Omega_{\text{mat}0} \approx 0.27 \) and \( H_0 = 100h \) km s\(^{-1}\)Mpc\(^{-1}\) with \( h \approx 0.71 \).

The molecular weight \( \mu_e \) is assumed to be the value

\[
\mu_e = \frac{1}{1 - Y/2}
\]

the primordial helium abundance \( Y \) to be \( Y \approx 0.25 \)

Assuming that the black hole seed forms from the collapse of a first generation, Population III star at redshift \( z \lesssim 40 \) and \( t \gtrsim t(40) = 0.067 \)Gyr, the available time for accretion is reduced to \( t_{\text{accrete}} \lesssim 0.80 \)Gyr.

Wagoner 1969, Omukai & Palla 2003: the stellar evolution (hydrogen-burning) lifetime of a massive Population III progenitor, \( t_{\text{env}} \sim 0.003 \)Gyr \( \ll t(40) \approx 0.067 \)Gyr. -The delay between stellar formation and collapse is of little consequence for determining the total time available for accretion growth.

The exponential accretion growth timescale is

\[
\tau_{\text{growth}} = \frac{\epsilon_M}{\epsilon_L (1 - \epsilon_M)} = 0.0394 \frac{\epsilon_M}{\epsilon_L (1 - \epsilon_M)} \text{Gyr \ (1.18)}
\]

which is considerably smaller than \( t_{\text{accrete}} \).

Cosmological Model \( \Lambda \)CDM, spatially flat model

\[
\left( \frac{\dot{a}}{a} \right)^2 = H_0^2 \left[ \frac{\Omega_{\text{mat}0}}{a^3} + \Omega_{\Lambda0} \right]
\]

where \( \Omega_{\text{mat}0} + \Omega_{\Lambda0} = 1 \), then we obtain

\[
t(z) = \frac{2}{3H_0 \sqrt{1 - \Omega_{\text{mat}0}}} \sinh^{-1} \left[ \frac{1 - \Omega_{\text{mat}0}}{\Omega_{\text{mat}0}} \left( 1 + z \right)^{3/2} \right]
\]
After the initial transient lasting $\sim 0.1\tau_{\text{acc}}$, during which time the black hole grows by a factor of $\sim 2$, the black hole spin and efficiency approach their asymptotic values.

(1.9) with (1.12), holding $\epsilon_M$ fixed, gives

$$\tau_{\text{spin}} = \frac{\epsilon_M \tau_{\text{acc}}}{\epsilon_L} \approx \frac{1 - \epsilon_M}{3.30} \tau_{\text{growth}},$$

which explains the rapid spin-up rate.

3 Cosmological implication

The luminosity is assumed to be the Eddington value ($\epsilon_L = 1$) and (1.13). The range of equilibrium accretion disk radiation efficiencies required to achieve the necessary growth of a black hole seed to supermassive size by $z_f = 6.43$ is $\epsilon_M \approx 0.13$. This requirement is equivalent to $a_\ast \approx 0.83$.

Therefore, it is likely that mergers are required to assist accretion to achieve black hole growth to supermassive size by $z_f = 6.43$.

Monte Carlo simulations by Yoo & Miralda-Escudé (2004) of hierarchical CDM halo mergers, accompanied by mergers of their central black holes, suggest that black hole mass amplification factors of $f \sim 10^4$.

In the major merger case or the minor one, after a short transient epoch, accretion will drive the merged remnant to the disk accretion equilibrium spin rate and corresponding mass efficiency.

The range of equilibrium accretion disk radiation efficiencies required to achieve the necessary growth of a black hole seed to supermassive size by $z_f = 6.43$ is consistent with the values inferred observationally for $R$, the range of $\epsilon_M$ favoring accretion disk models that drive the black hole to spin equilibrium in the range $0.7 \lesssim a_\ast \lesssim 0.95$.

Set $t_i = 0$, $z_i = \infty$ so that the plotted radiation efficiency represents an upper limit to $\epsilon_M$.

In the absence of mergers, the upper limit to the $\epsilon_M$ required to build an SMBH by $z_{\text{QSO}} \approx 6.43$ is $\epsilon_M \approx 0.14$. When mergers are included, the upper limit to the efficiency increases to $\epsilon_M \sim 0.30$.

- Should a quasar be discovered at $z_{\text{QSO}} > 6.43$, it would appear that accretion from a standard thin disk will be ruled out.

- Should a quasar be discovered at $z_{\text{QSO}} > 10$, $\epsilon_M$ would fall below 0.19 (=$\epsilon_M(0.95)$) and the results would be difficult to reconcile with accretion from a typical MHD disk as modeled in recent simulations.

- Should a quasar be discovered at $z_{\text{QSO}} > 18$, the upper limit to $\epsilon_M$ would drop below the observationally inferred value 0.1 for all $z_i > z_{\text{QSO}}$. 